# Quadratic Graphs

Quadratic graphs have equations of the form:

$y = ax^2 + bx + c$ where $a$, $b$, $c$ are positive or negative constants
($b$ and/or $c$ could also be zero)

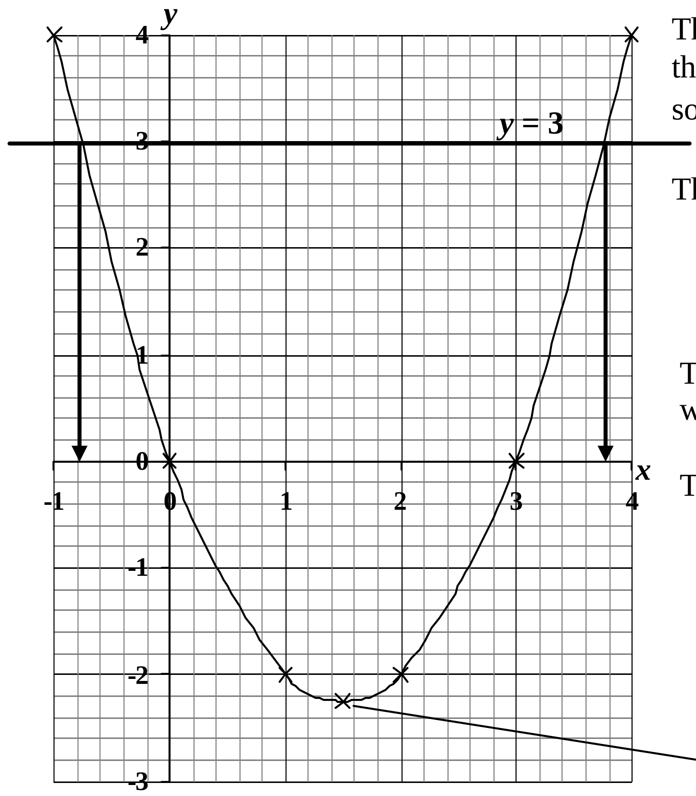
To draw a quadratic graph from its equation, you need to calculate and plot points.
You need to plot enough points to give the shape of the curve.

**Example** $y = x^2 - 3x$ gives these points:

| | | | | | | |
|---|---|---|---|---|---|---|
| ***x*** | – 1 | 0 | 1 | 2 | 3 | 4 |
| ***y*** | 4 | 0 | – 2 | – 2 | 0 | 4 |

In this case it is useful to work out an extra point:
When $x = 1.5$, $y = 1.5^2 - 3 \times 1.5 = -2.25$ This is the lowest point on the curve.

**Graph of $y = x^2 - 3x$**




The curve crosses the $y$ axis at (0, 0) and the $x$ axis at (0, 0) and (3, 0).

The points where the curve crosses the $x$ axis (i.e. the line $y = 0$) give the solutions of the equation $x^2 - 3x = 0$.

The solutions of $x^2 - 3x = 0$ are:
***x* = 0** and **3**

The solutions of $x^2 - 3x = 3$ are found where the curve crosses the line $y = 3$.

The solutions are ***x* = – 0.8** and **3.8**
(correct to 1 decimal place)

The curve has a ***minimum point*** at **(1.5, – 2.25).**

All quadratic equations $y = ax^2 + bx + c$ have this characteristic shape.
When ***a* is positive**, the curve has a ***minimum point*** like this one.
When ***a* is negative**, the curve is the other way up and has a ***maximum point***.



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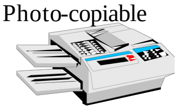


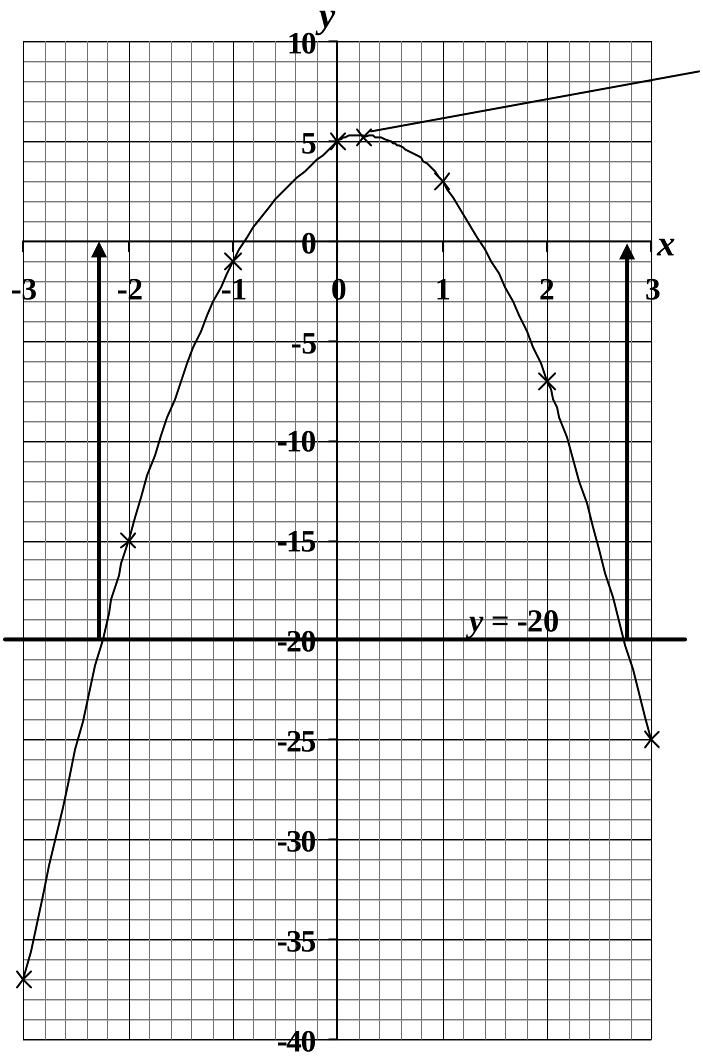
**Example** $y = 5 + 2x - 4x^2$ gives the following points:

| $x$ | $-3$ | $-2$ | $-1$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | $-37$ | $-15$ | $-1$ | 5 | 3 | $-7$ | $-25$ |

To help get the shape right near the highest point, it is useful to work out extra points:
eg when $x = 0.5$, $y = 5 + 2 \times 0.5 - 4 \times 0.5^2 = 5$
when $x = 0.25$, $y = 5 + 2 \times 0.25 - 4 \times 0.25^2 = 5.25$ (the maximum value of $y$)

The graph is shown below.

**Graph of $y = 5 + 2x - 4x^2$**




The ***maximum point*** on this curve is **(0.25, 5.25).**

This curve crosses the $y$ axis at (0, 5) and the $x$ axis at (– 0.9, 0) and (1.4, 0) (correct to 1 decimal place)

The points where the curve crosses the $x$ axis give the solutions of the equation

$$5 + 2x - 4x^2 = 0$$

The solutions are ***x* = – 0.9** and **1.4**
(correct to 1 decimal place)

The graph can be used to solve other equations.

For example $25 + 2x - 4x^2 = 0$ is equivalent to $5 + 2x - 4x^2 = -20$ so look for the $x$ values where the curve crosses $y = -20$

The solutions are ***x* = – 2.3** and **2.8**
(correct to 1 decimal place)



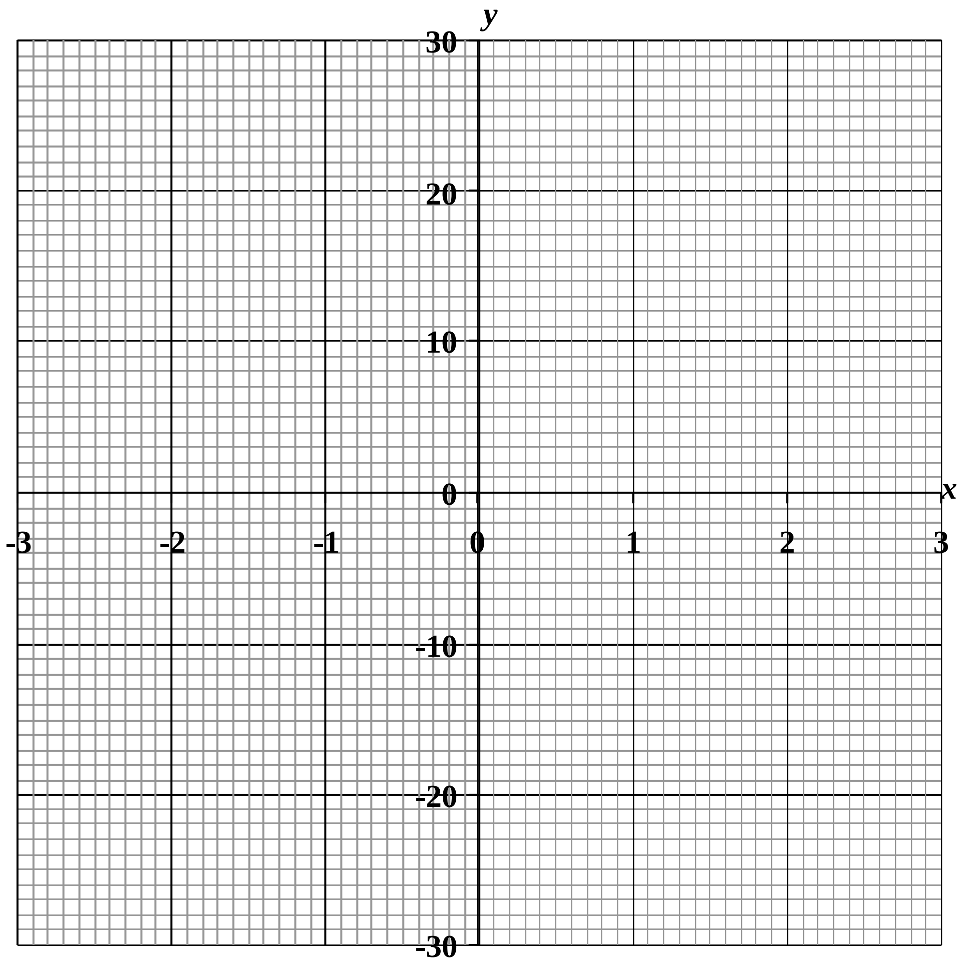
***Try these….***

1 a) Complete the table:

| $x$ | $-3$ | $-2$ | $-1$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $x^2$ | | | | | | | |
| $2x^2$ | | | | | | | |
| $3x^2$ | | | | | | | |
| $-x^2$ | | | | | | | |
| $-2x^2$ | | | | | | | |
| $-3x^2$ | | | | | | | |

b) On the grid below draw and label the graphs of the following:

$y = x^2$ $y = 2x^2$ $y = 3x^2$ $y = -x^2$ $y = -2x^2$ $y = -3x^2$




c) Write down what you notice about your graphs.

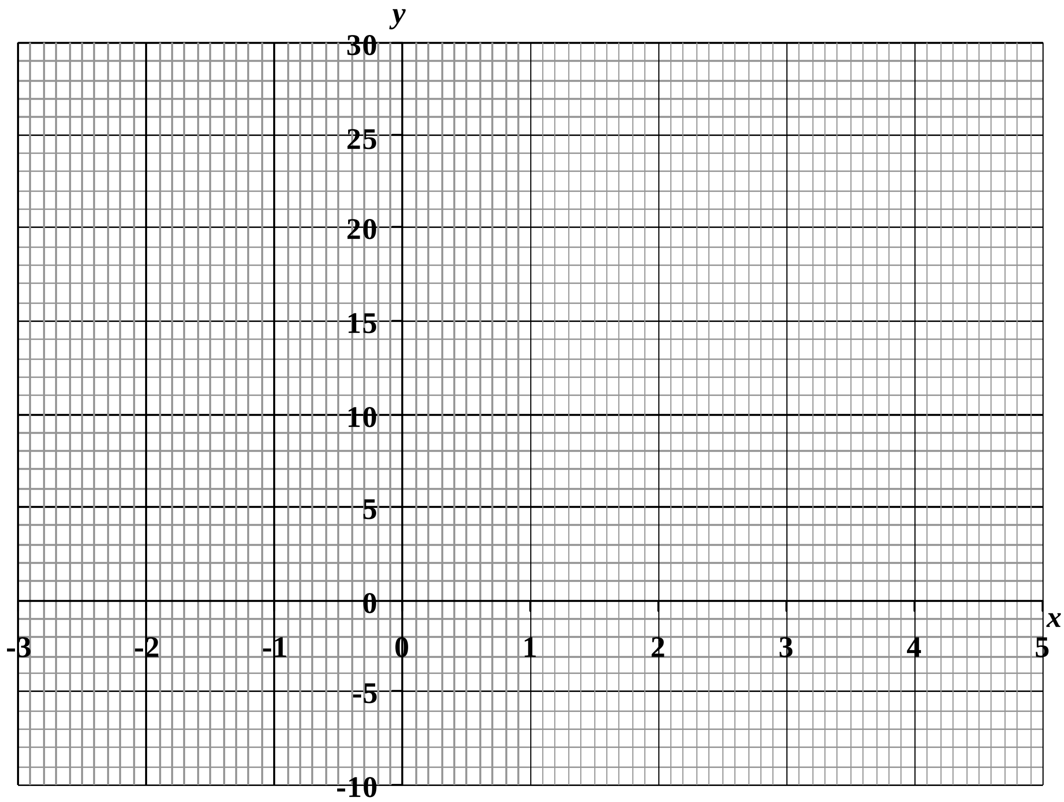
2 a) Complete the table below for $y = 2x^2 - 5x - 3$

| ***x*** | $-3$ | $-2$ | $-1$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|---|---|---|
| ***y*** | | | | | | | | | |

b) On the grid below plot the points from the table, but do not join them yet.

c) Find the value of $y$ when $x = 1.25$ and plot this point on the grid.

**Graph of $y = 2x^2 - 5x - 3$**




d) Join the points with a smooth curve.

e) Use your graph to solve the following equations:

(i) $2x^2 - 5x - 3 = 0$

(ii) $2x^2 - 5x - 3 = 5$

(iii) $2x^2 - 5x - 3 = 19$

(iv) $2x^2 - 5x = 0$

(v) $2x^2 - 5x + 2 = 0$

(vi) $2x^2 - 5x - 9 = 0$

3 a) Draw the graph of $y = 4x - x^2$ for values of $x$ between – 1 and 5
b) What is the maximum point on the curve?
c) Use your graph to solve the following equations:

(i) $4x - x^2 = 0$ (ii) $4x - x^2 = 2$

(iii) $4x - x^2 = -3$ (iv) $4 + 4x - x^2 = 0$

4 a) Draw the graph of $y = x^2 - 3x - 4$ for values of $x$ between – 2 and 5
b) Find the coordinates of the minimum point on the curve.
c) Use your graph to solve the following equations:

(i) $x^2 - 3x - 4 = 0$ (ii) $x^2 - 3x - 4 = 3$

(iii) $x^2 - 3x - 4 = -2$ (iv) $x^2 - 3x = 0$

(v) $x^2 - 3x + 1 = 0$ (vi) $x^2 - 3x - 6 = 0$

5 a) Draw the graph of $y = 3x^2 + 2x - 7$ for values of $x$ between – 4 and 3
b) Give approximate coordinates for the minimum point on the curve.
c) Use your graph to solve the following equations:

(i) $3x^2 + 2x - 7 = 0$ (ii) $3x^2 + 2x - 7 = 20$

(iii) $3x^2 + 2x - 7 = -5$ (iv) $3x^2 + 2x - 17 = 0$

(v) $3x^2 + 2x = 0$

d) Explain how you can tell from the graph that the equation $3x^2 + 2x + 3 = 0$ has no solutions.

6 a) Draw the graph of $y = 9 - 2x - 2x^2$ for values of $x$ between – 4 and 3
b) Estimate the coordinates of the maximum point on the curve.
c) Use your graph to solve the following equations:

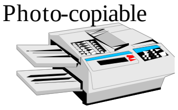
(i) $9 - 2x - 2x^2 = 0$ (ii) $9 - 2x - 2x^2 = 7$

(iii) $9 - 2x - 2x^2 = -12$ (iv) $2x^2 + 2x = 0$

(v) $2x^2 + 2x = 5$ (vi) $2x^2 + 2x - 19 = 0$

d) Explain how you can tell from the graph that the equation $2x^2 + 2x + 3 = 0$ has no solutions.





## Teacher Notes

**Unit** Intermediate Level, Using algebra, functions and graphs

**Skills used in this activity:**
- Drawing graphs of quadratic functions
- Using graphs to find the solutions to quadratic equations.

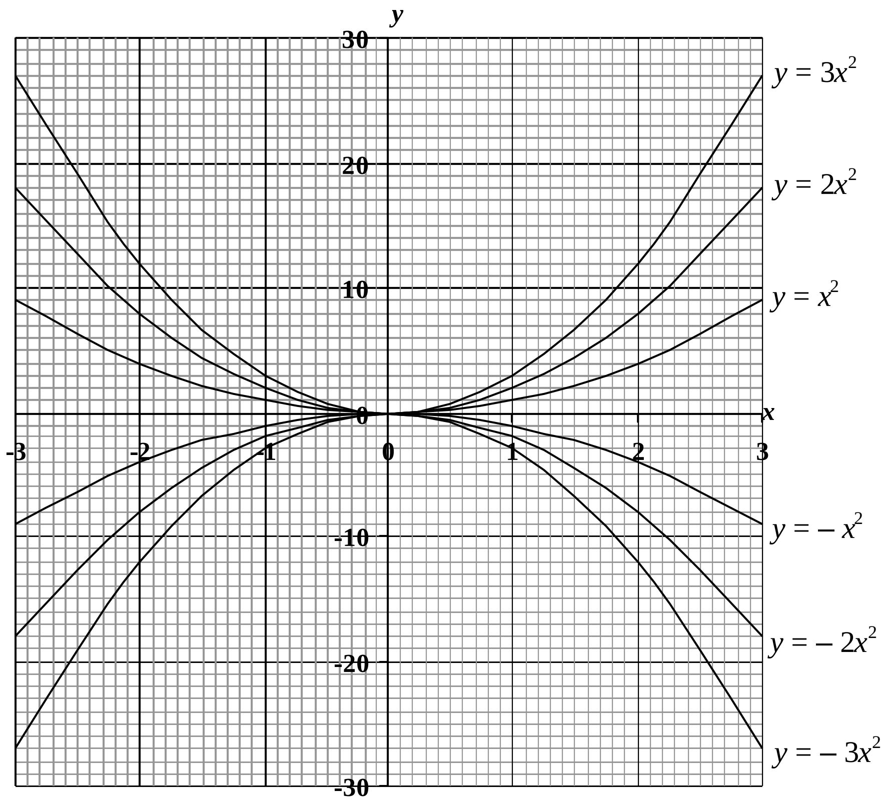
**Notes**
This activity can be used to introduce quadratic graphs or as a revision exercise at the end of the course. The accompanying Powerpoint presentation includes the examples that are given on pages 1 and 2. The questions on pages 2 and 3 can be done on the worksheet, but those on page 5 expect students to draw the graphs on graph paper. Alternatively, students can use graphic calculators to answer all of the questions.

**Answers**

1 a)

| $x$ | – 3 | – 2 | – 1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $x^2$ | 9 | 4 | 1 | 0 | 1 | 4 | 9 |
| $2x^2$ | 18 | 8 | 2 | 0 | 2 | 8 | 18 |
| $3x^2$ | 27 | 12 | 3 | 0 | 3 | 12 | 27 |
| $-x^2$ | – 9 | – 4 | – 1 | 0 | – 1 | – 4 | – 9 |
| $-2x^2$ | – 18 | – 8 | – 2 | 0 | – 2 | – 8 | – 18 |
| $-3x^2$ | – 27 | – 12 | – 3 | 0 | – 3 | – 12 | – 27 |

b)




c) Possible answers include:
All curves pass through the origin. As the coefficient of $x^2$ increases, the curve becomes steeper.
Positive $x^2$ terms give a $\cup$ shaped curve, whilst negative $x^2$ terms give a $\cap$ shaped curve.

2 a)

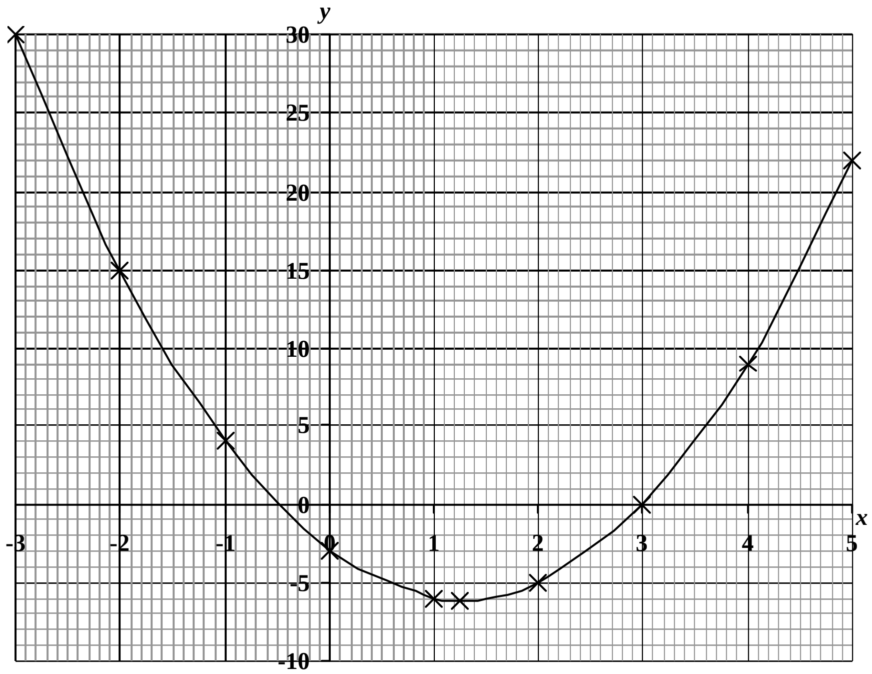
| $x$ | – 3 | – 2 | – 1 | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|---|---|---|
| $y$ | 30 | 15 | 4 | – 3 | – 6 | – 5 | 0 | 9 | 22 |

b) see graph below

c) - 6.125

d)

**Graph of $y = 2x^2 - 5x - 3$**




e) (i) - 0.5, 3

(ii) - 1.1, 3.6

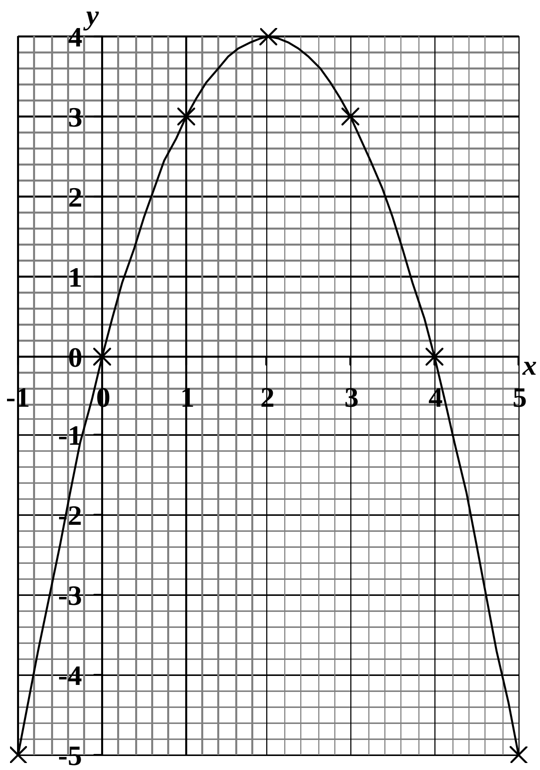
(iii) - 2.3, 4.8

(iv) 0, 2.5

(v) - 1.2, 3.7

3 a)

**Graph of $y = 4x - x^2$**




b) (2, 4)

c) (i) 0, 4

(ii) 0.6, 3.4

(iii) - 0.6, 4.6

(iv) - 0.8, 4.8



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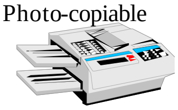
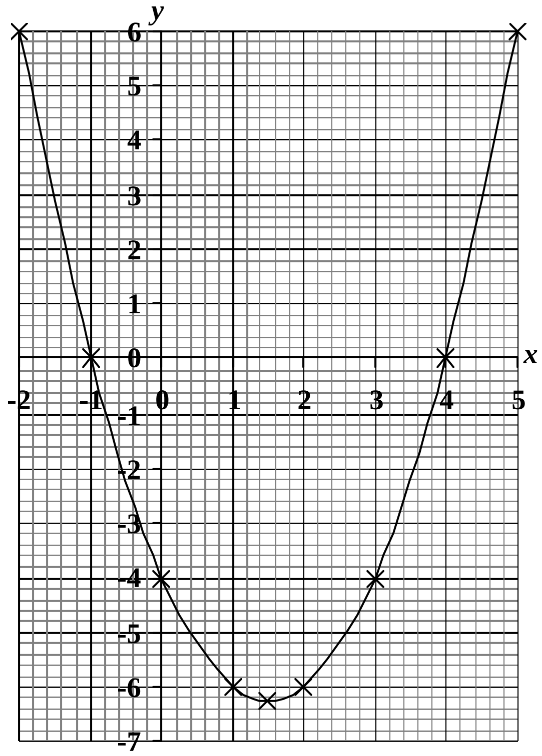


4 a) **Graph of $y = x^2 - 3x - 4$**




b) (1.5, - 6.25)

c) (i) -1, 4

(ii) - 1.5, 4.5

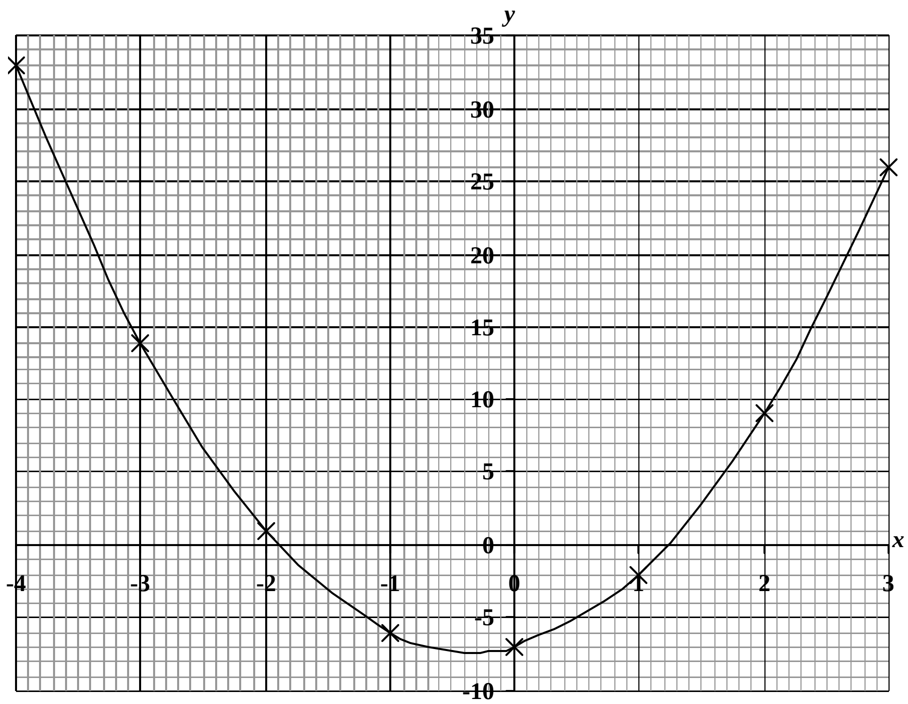
(iii) - 0.6, 3.6

(iv) 0, 3

(v) 0.4, 2.6

(vi) - 1.4, 4.4

5 a) **Graph of $y = 3x^2 + 2x - 7$**




b) (-0.3, - 7.3)

c) (i) - 1.9, 1.2

(ii) - 3.3, 2.7

(iii) - 1.2, 0.5

(iv) - 2.7, 2.1

(v) - 0.7, 0

d) $3x^2 + 2x + 3 = 0$ is equivalent to $3x^2 + 2x - 7 = -10$ .
The curve does not cross the line $y$ = - 10, so there are no solutions.



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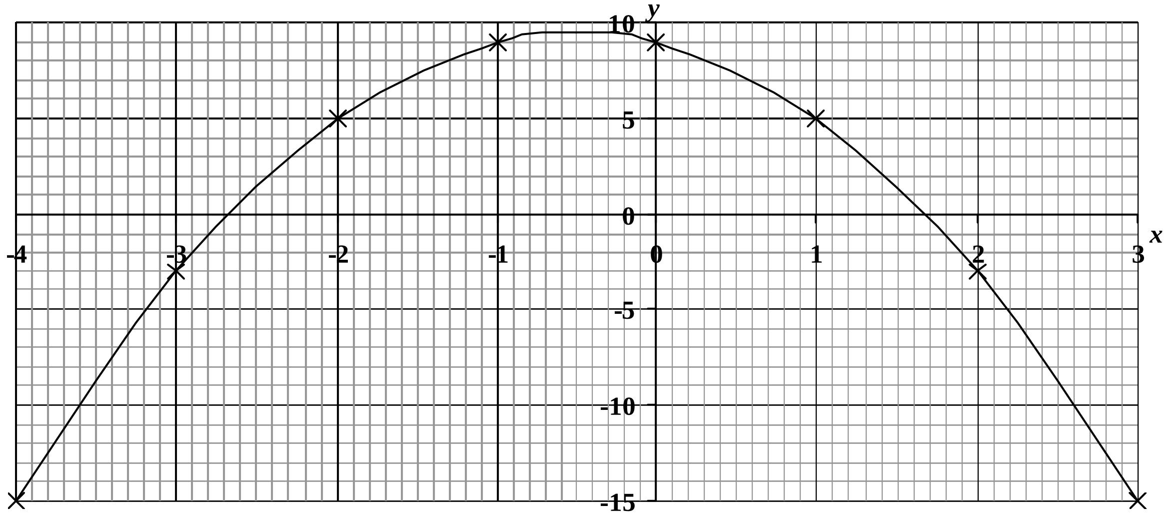
6 a)

**Graph of $y = 9 - 2x - 2x^2$**




b) (- 0.5, 9.5)

c) (i) -2.7, 1.7 (ii) - 1.6, 0.6 (iii) - 3.8, 2.8

(iv) -1, 0 (v) -2.2, 1.2 (vi) - 3.6, 2.6

d) $2x^2 + 2x + 3 = 0$ is equivalent to $9 - 2x - 2x^2 = 12$.
The curve does not cross the line $y = 12$, so there are no solutions.